Multi-Invariance ESPRIT-Based Blind DOA Estimation for MC-CDMA With an Antenna Array

Xiaofei Zhang, Xin Gao, and Dazhuan Xu

Abstract—In this paper, we address the problem of direction-of-arrival (DOA) estimation for a multicarrier code-division multiple-access (MC-CDMA) system with an antenna array. We reconstruct the received signal to form a data model with a multi-invariance property, and then, a multi-invariance estimation of signal parameters via rotational invariance technique (ESPRIT) algorithm for DOA estimation is proposed. This algorithm has improved DOA estimation performance and identified more DOAs compared with an ESPRIT algorithm. Moreover, our algorithm enables DOA estimation of a large number of impinging waves. Simulation results illustrate the performance of this algorithm.

Index Terms—Antenna array, direction-of-arrival (DOA) estimation, estimation of signal parameters via rotational invariance techniques (ESPRIT), multicarrier code-division multiple access (MC-CDMA), multi-invariance.

Manuscript received April 28, 2008; revised October 5, 2008, December 15, 2008, and February 18, 2009. First published April 10, 2009; current version published October 29, 2009. This work was supported in part by China National Science Foundation (NSF) under Grant 60801052, by the Ph.D. Programs Foundation of the Ministry of Education of China under Grant 200802871056, and by Jiangsu NSF under Grant BK2007192. The review of this paper was coordinated by Dr. T. Taniguchi.

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Digital Object Identifier 10.1109/TVT.2009.2020596

1. INTRODUCTION

Recently, multicarrier code-division multiple access (MC-CDMA) [1], [2] has received extensive attention in the context of fourth-generation (4G) communication systems [3]. MC-CDMA has its capabilities of achieving high capacity over a frequency-selective fading channel [4]–[6]. An antenna array in an MC-CDMA base station exploits the spatial domain to provide an extra way of cochannel interference cancellation and thus tends to improve system capacity [7]–[9]. Blind direction-of-arrival (DOA) estimation methods for an array antenna MC-CDMA system contain estimation of signal parameters via rotational invariance techniques (ESPRIT) [10], [11] and multiple signal classification (MUSIC) [12]. In contrast to training-based methods, the blind DOA estimators improve bandwidth (BW) efficiency. Notably, the ESPRIT method [10], [11] exploits the inherent shift-invariant structure of the received signal and has high-accuracy estimation performance. In this paper, we reconstruct the received signal to form a data model with a multi-invariance property and then derive a multi-invariance ESPRIT algorithm for DOA estimation. Compared with the ESPRIT algorithm, our proposed algorithm has improved DOA estimation performance and identifies more DOAs than ESPRIT. It also enables DOA estimation of a large number of impinging waves.

The remainder of this paper is structured as follows. Section II develops a data model, whereas Section III derives a multi-invariance ESPRIT algorithm. Error analysis is presented in Section IV. Sections V and VI offer our simulation results and conclusions, respectively.

Notation: We denote complex conjugation by $(\cdot)^\ast$, matrix transpose by $(\cdot)^T$, and matrix conjugate transpose by $(\cdot)^{H}$. The notation $(\cdot)^{\dagger}$ refers to the Moore–Penrose inverse (pseudoinverse). real$(\cdot)$ is to get the real part of a complex number.

II. DATA MODEL

Let us assume there are $K$ users in the MC-CDMA system, and the receiver is equipped with a uniform linear array (ULA) containing $I$ antennas. The transmitter structure of the $k$th user is shown in Fig. 1. The symbol sequence of the $k$th user is $b_k = [b_k(1), b_k(2), \ldots, b_k(L)]^T$. The data symbol $b_k$ is a BPSK-modulated signal and transmitted in parallel over $N$ subcarriers, each multiplied by a different element of the spread sequence $c_k$, where $c_k = [c_k(1), c_k(2), \ldots, c_k(N)]^T$ is the spread code of the $k$th user. The output signal of the spread spectrum is shown as $U_k = c_k b_k^T$, and the signal $U_k$ is processed under multicarrier modulation, which can also be denoted by inverse fast Fourier transform (IFFT). Hence, the output signal of multicarrier modulation is hereby expressed as

$$ D_k = F^H U_k = F^H c_k b_k^T $$

Fig. 1. Transmitter structure of the $k$th user.
where $\mathbf{F}$ is the fast Fourier transform matrix with $N \times N$. $\mathbf{F}^H$ stands for the IFFT.

We also assume that each of the $K$ users has only a single path, through which they synchronously arrive at the antenna array. The array spacing is $d = \lambda_c/2 = c/2f_c$, where $f_c$, $\lambda_c$, and $c$ are the carrier frequency, wavelength, and light speed, respectively. Meanwhile, we define the $n$th subcarrier frequency $f_n = f_c + (n-1)\Delta f$, where $\Delta f$ is the subcarrier spacing. Considering an MC-CDMA system with a carrier frequency of 5 GHz, 32 subcarriers, and a subcarrier spacing of 25 kHz, we suppose that the signal of the $4$th user impinges the array antenna with angle $\theta_k$, and the channel response between the $n$th subcarrier of the $4$th user and the $n$th antenna should be

$$h_{i,k,n} = \exp(-j2\pi(i-1)df_n \sin \theta_k/c) h_{i,k,n} = \exp(-j\pi(i-1) \sin \theta_k)$$

$$\times \exp \left(-j2\pi(i-1)(n-1)\frac{\Delta f}{2f_c} \sin \theta_k \right) h_{i,k,n}$$

where $j = \sqrt{-1}$, and $h_{i,k,n}$ is the channel response of the $n$th carrier of the $i$th user and the first antenna. Compared with the carrier frequency $f_c$, the subcarrier spacing $\Delta f$ is extremely small. In addition, the carrier frequency $f_c$ is much larger than the transmission BW. In general, $\Delta f/f_c < 0.0001$ and $BW/f_c < 0.01$; for example, $f_c = 5$ GHz, $BW = 800$ kHz, and $\Delta f = 25$ kHz in this paper. It is approximately estimated that $\exp(-j2\pi(i-1)(n-1)(\Delta f/2f_c) \sin \theta_k) \cong 1$, and (2) becomes

$$h_{i,k,n} = \exp(-j\pi(i-1) \sin \theta_k) h_{i,k,n}, \quad n = 1, 2, \ldots, N.$$  (3)

According to (3)

$$\begin{bmatrix} h_{1,k,n} \\ h_{2,k,n} \\ \vdots \\ h_{I,k,n} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j\pi \sin \theta_k} \\ \vdots \\ \vdots \\ e^{-j\pi(1-1) \sin \theta_k} \end{bmatrix} h_{i,k,n}, \quad n = 1, 2, \ldots, N.$$  (4)

Although each subcarrier has a different frequency, different subcarriers of a user have the same array response, which was also assumed in [8]–[14]. Considering the subcarrier signals of a user pass through the same channel, different subcarriers of a user toward a receive antenna have the same channel response. That is to say, $h_{i,k,n} = h_{i,k}, n = 1, 2, \ldots, N$, where $h_{i,k}$ is the channel response between the $k$th user and the $i$th antenna.

We assume $K$ users impinging the array antenna, and the received signal of the $i$th antenna is

$$\begin{align*}
X_i &= \sum_{k=1}^{K} D_k h_{i,k} = \sum_{k=1}^{K} \mathbf{F}^H e_k b_k^T h_{i,k} \\
&= \mathbf{F}^H C\text{diag}\{h_{i,1}, h_{i,2}, \ldots, h_{i,K}\} \mathbf{B}^T, \quad i = 1, 2, \ldots, I
\end{align*}$$

(4)

where $C = [e_1, e_2, \ldots, e_K] \in \mathbb{R}^{N \times K}$ is the spread matrix, and $\mathbf{B} = [b_1, b_2, \ldots, b_K] \in \mathbb{R}^{L \times K}$ is the source matrix.

Define the channel matrix $\mathbf{H}$ by

$$\mathbf{H} = \begin{bmatrix} h_{i,1} & h_{i,2} & \cdots & h_{i,K} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{I,1} & h_{I,2} & \cdots & h_{I,K} \end{bmatrix} \in \mathbb{C}^{I \times K}$$

$$h_{i,k} = h_{i,k} e^{-j\pi(i-1) \sin \theta_k}.$$  (5)

Equation (5) is also denoted by $\mathbf{H} = \mathbf{A} \Phi$, where $\Phi = \text{diag}\{h_{1,1}, h_{1,2}, \ldots, h_{1,K}\}$, $\mathbf{A} = [a(\theta_1), a(\theta_2), \ldots, a(\theta_K)]$ is the direction matrix, and $a(\theta_k) = [1, e^{-j\pi \sin \theta_k}, \ldots, e^{-j\pi(1-1) \sin \theta_k}]^T$.

Equation (4) is also denoted by

$$X_i = \mathbf{S} \mathbf{D}_i(\mathbf{H}) \mathbf{B}^T, \quad i = 1, 2, \ldots, I$$

(6)

where $\mathbf{S} = \mathbf{F}^H C \in \mathbb{C}^{N \times K}$, and $\mathbf{D}_i(.)$ is to extract the $i$th row of its matrix and construct a diagonal matrix out of it. In the presence of noise, the received signal model becomes $\mathbf{X}_i = \mathbf{S} \mathbf{D}_i(\mathbf{H}) \mathbf{B}^T + \mathbf{V}_i$, where $\mathbf{V}_i$ is the received noise corresponding to the $i$th antenna.

In addition, (6) can be regarded as a trilinear model [15], which also has another matrix system rearrangement way, such as $\mathbf{Y}_i = \mathbf{H} \mathbf{D}_i(\mathbf{B}) \mathbf{S}^T = \mathbf{A} \mathbf{F} \mathbf{D}_i(\mathbf{B}) \mathbf{S}^T, i = 1, 2, \ldots, I$, in which the matrix $\mathbf{A}$ is with Vandermonde characteristic, and then, the ESPRIT algorithm [16] can be used for DOA estimation.

III. MULTI-INVARIENCE ESPRIT-BASED DOA ESTIMATION ALGORITHM

According to (6), we form the following matrix:

$$\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_I \end{bmatrix} = \begin{bmatrix} \mathbf{S} \mathbf{D}_1(\mathbf{H}) \\ \mathbf{S} \mathbf{D}_2(\mathbf{H}) \\ \vdots \\ \mathbf{S} \mathbf{D}_I(\mathbf{H}) \end{bmatrix} \mathbf{B}^T$$

$$= \begin{bmatrix} \mathbf{S} \\ \mathbf{S} \mathbf{\Phi} \\ \vdots \\ \mathbf{S} \mathbf{\Phi}^{I-1} \end{bmatrix} \mathbf{B}^T$$

(7)

where $\mathbf{B}_E = \mathbf{B} \Phi = \text{diag}\{e^{-j\pi \sin \theta_1}, e^{-j\pi \sin \theta_2}, \ldots, e^{-j\pi \sin \theta_K}\} \in \mathbb{C}^{K \times K}$ is called the rotation matrix. According to the multi-invariance characteristic of the signal in (7), we use multi-invariance ESPRIT [17] to estimate DOAs. For (7), $\mathbf{R} = \mathbf{X} \mathbf{X}^H$. We denote the matrix containing the eigenvectors of $\mathbf{F}^H \mathbf{F}$ associated with the $K$ largest eigenvalues of $\mathbf{R}$ by $\mathbf{E}$, i.e.,

$$\mathbf{E} = \begin{bmatrix} \mathbf{S} \\ \mathbf{S} \mathbf{\Phi} \\ \vdots \\ \mathbf{S} \mathbf{\Phi}^{I-1} \end{bmatrix} \mathbf{T}$$

(8)

where $\mathbf{T}$ is a $K \times K$ full-rank matrix. According to (8), we define $\mathbf{E}_1$ and $\mathbf{E}_2$ by

$$\mathbf{E}_1 = \begin{bmatrix} \mathbf{S} \\ \mathbf{S} \mathbf{\Phi} \\ \vdots \\ \mathbf{S} \mathbf{\Phi}^{I-2} \end{bmatrix} \mathbf{T}$$

$$\mathbf{E}_2 = \begin{bmatrix} \mathbf{S} \mathbf{\Phi} \\ \vdots \\ \mathbf{S} \mathbf{\Phi}^{I-2} \end{bmatrix} \mathbf{T}.$$  (9)

According to (9)

$$\mathbf{E}_2 = \begin{bmatrix} \mathbf{S} \\ \mathbf{S} \mathbf{\Phi} \\ \vdots \\ \mathbf{S} \mathbf{\Phi}^{I-2} \end{bmatrix} \mathbf{T} = \begin{bmatrix} \mathbf{S} \mathbf{\Phi} \\ \vdots \\ \mathbf{S} \mathbf{\Phi}^{I-2} \end{bmatrix} \mathbf{T} = \mathbf{T}.$$  (10)

Define $\mathbf{\Omega} = \mathbf{T}^{-1} \mathbf{\Phi} \mathbf{T}$. Equation (10) becomes $\mathbf{E}_2 = \mathbf{E}_1 \mathbf{\Omega}$, and then $\mathbf{\Omega} = \mathbf{E}_1 \mathbf{\Omega}$. Because $\mathbf{\Omega}$ has the same eigenvalues as $\mathbf{\Phi}$, we use eigenvalue decomposition for $\mathbf{\Omega}$ to get $e^{-j\pi \sin \theta_k}, k = 1, 2, \ldots, K$, and then estimate DOA $\theta_k, k = 1, 2, \ldots, K$.

It should be pointed out that the ESPRIT algorithm only works well for $K < I$, where $K$ and $I$ are the numbers of users and antennas,
respectively. Our proposed algorithm does not have this constraint. Exploiting the multiple invariance characteristic in (9) and (10), it is easy to determine the maximum number of users $K_{\text{max}}$ that our proposed algorithm can detect. It is clear that $K_{\text{max}} = N(1 - 1)$, $\forall I \geq 2$ (in general, $N > I$), when the matrix $S \in \mathbb{C}^{N \times k}$ is full rank. We also consider that, if $K > N$, the matrix $S$ is rank deficient. Hence, the maximum number of users $K_{\text{max}}$ is $N$, or $K \leq N$.

### IV. Error Analysis

This section aims at analyzing errors, including the array gain error, array phase error, model error, and mutual coupling. Considering the array gain error and array phase error, the direction matrix becomes $\tilde{A} = A + \delta A$, where $\delta A$ is the random array error matrix. Then, (8) becomes

$$
\begin{bmatrix}
S D_1 (A + \delta A) \\
S D_2 (A + \delta A) \\
\vdots \\
S D_l (A + \delta A)
\end{bmatrix}
= 
\begin{bmatrix}
S \\
S \Psi \\
\vdots \\
S \Psi^{l-2}
\end{bmatrix}
\begin{bmatrix}
\partial A_1 \\
\partial A_2 \\
\vdots \\
\partial A_l
\end{bmatrix}
T.
$$

In addition, we get $E_1$ and $E_2$, followed by

$$
E_1 = 
\begin{bmatrix}
S D_1 (A + \delta A) \\
S D_2 (A + \delta A) \\
\vdots \\
S D_{l-1} (A + \delta A)
\end{bmatrix}
T = [A_1 + \partial A_1] T
$$

$$
E_2 = 
\begin{bmatrix}
S D_2 (A + \delta A) \\
S D_3 (A + \delta A) \\
\vdots \\
S D_l (A + \delta A)
\end{bmatrix}
T = [A_1 + \partial A_2] \Psi T
$$

where

$$
A_1 = 
\begin{bmatrix}
S D_1 (A) \\
S D_2 (A) \\
\vdots \\
S D_{l-1} (A)
\end{bmatrix}
= 
\begin{bmatrix}
S \\
S \Psi \\
\vdots \\
S \Psi^{l-2}
\end{bmatrix}
\begin{bmatrix}
\partial A_1 \\
\partial A_2 \\
\vdots \\
\partial A_l
\end{bmatrix}
T.
$$

According to the first-order approximation for $[A_1 + \partial A_1]^+$, we get

$$
\hat{\Omega} = T^{-1} \left( I + A_1^+ (\partial A_2 - \partial A_1) \right) \Psi T.
$$

The $k$th eigenvalue of $\hat{\Omega}$ is $\hat{\lambda}_k = p_k + \partial p_k$, where $\partial p_k = p_k e_k^T \partial A_2 - \partial A_1 e_k$, and $e_k$ is a unit vector, in which the $k$th element is 1, and others are zeros. Using the first-order Taylor series expansion, the variance of DOA estimation is

$$
E \left[ \partial \theta_k^2 \right] = \frac{1}{2} \frac{\lambda}{2 \pi d \cos \theta_k}^2 \left[ E \left[ |\partial p_k|^2 \right] - \text{real} \left[ E \left[ |\partial p_k|^2 |\partial p_k|^2 \right] \right] \right].
$$

For the existence of mutual coupling, the direction matrix should be modeled with $\tilde{A} = ZA$, where $Z$ is an $I \times I$ mutual coupling matrix. The mutual coupling coefficients between two elements that are far enough from each other can be approximated as zero, and the coupling between any two equally spaced sensors appears the same [18]. For an ULA, a mutual coupling matrix can be model as a banded symmetric Toeplitz matrix $Z$, which is given by

$$
\begin{cases}
z_{ik} = z_{i-k+1}, & 1 \leq i, k \leq I \\
z_{ik} = 1, & 1 \leq i \leq I
\end{cases}
$$

where $z_{ik}$ is the $(i, k)$ element of the matrix $Z$. $\hat{A} = ZA = A + \delta A$, where $\delta A = (Z - I)A$, and $I$ is an $I \times I$ identity matrix. As aforementioned, we take the same method to analyze its error.

For multicarrier modulation signals, when the subcarrier differs, the array response varies. However, we assume that different subcarriers of a user have the same response for the array antenna. In this paper, the channel response between the $n$th subcarrier of the $k$th user and the $i$th antenna is approximated by $\exp (-j \pi (i - 1) \sin \theta_k) b_{1,k,n}$, and the channel response error is $\exp (-j \pi (i - 1) \sin \theta_k) [1 - \exp (-j \pi (i - 1)(n - 1)(\Delta f / 2 f_s) \sin \theta_k)] b_{1,k,n}$, which results in an array model error. The direction matrix with this array model error is $\tilde{A} = A + \partial A$, where $\partial A$ is considered as the model error. When dealing with a model error, we also take the same method similarly to analyze this kind of array gain error and array phase error.

### V. Simulation Results

Let the received noisy signal $\tilde{X}_i = S D_i (H) B^T + V_i$, $i = 1, 2, \ldots, I$, where $V_i$ is the additive Gaussian white noise matrix. In addition, we define SNR as

$$
\text{SNR} = 10 \log_{10} \frac{\sum_{i=1}^{I} \| S D_i (H) B^T \|^2}{\sum_{i=1}^{I} \| V_i \|^2} \text{ dB}.
$$

The MC-CDMA receiver is equipped with an eight-element ULA. Mutual coupling in the antenna array is neglected in this simulation. We hereby adopt BPSK symbols as the transmitted data, which are spread by Walsh–Hadamard sequences. The number of subcarriers or spread gain is 32. Note that $L$ is the number of snapshots, and $K$ is the number of users.

There are three MC-CDMA signals impinging on the ULA at $\theta_1 = 10^\circ$, $\theta_2 = 20^\circ$, and $\theta_3 = 30^\circ$, respectively. We compare our proposed algorithm with the ESPRIT algorithm.

Define RMSE $= \sqrt{(1/1000) \sum_{m=1}^{1000} |\theta_m - \theta_0|^2}$, where $\theta_0$ and $\theta_m$ are the perfect DOA and the estimated DOA of the $m$th Monte Carlo trial, respectively. Fig. 2 presents DOA estimation performance with $K = 3$ and $L = 50$. In Fig. 2, we find that our proposed algorithm works well. Fig. 3 shows DOA $= 10^\circ$ estimation performance comparison with $K = 3$ and $L = 50$. In Fig. 3, we find that our proposed algorithm has much better DOA estimation performance than the ESPRIT algorithm, and the performance of multi-invariance ESPRIT is very close to that of the Cramer–Rao bound.

Fig. 4 presents DOA $= 30^\circ$ estimation performance with $K = 3$ and different $L$. In Fig. 4, we find that the DOA estimation performance of our proposed algorithm is improved with $L$ increasing.

Fig. 5 shows DOA $= 20^\circ$ estimation performance with $L = 50$ and different $K$. In Fig. 5, we find that the DOA estimation performance of our proposed algorithm is improved with $K$ decreasing.

Array antennas contain $I = 8$ elements in this simulation, and the ESPRIT algorithm can identify $K = 7$ sources. When $K \geq I$ ($I$ and $K$ are the numbers of sources and antennas, respectively), the ESPRIT...
algorithm fails to work. Our proposed algorithm can identify 32 source signals in this simulation. Suppose there are 30 signals impinging on an ULA at $\theta = [0^\circ : 3^\circ : 87^\circ]$, respectively. Fig. 6 shows DOA estimation performance with $K = 30$ and $L = 100$. In Fig. 6, we conclude that our proposed algorithm works well under the condition of a larger user number. We also find that DOA estimation performance is improved with the value of DOA decreasing.

Practically, the array response error caused by multicarrier modulation brings about some negative influence on our proposed DOA estimation. This kind of influence is quite little and can even be neglected when $\Delta f < < f_c$; as $\Delta f / f_c$ rises, the influence that was introduced also gradually worsens and renders decreasing DOA estimation performance.

VI. Conclusion

Multi-invariance ESPRIT-based DOA estimation for an MC-CDMA system with an antenna array has been proposed. This algorithm has improved DOA estimation performance and can identify more DOAs than the ESPRIT method. Furthermore, our algorithm enables DOA estimation of a large number of impinging waves.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their valuable suggestions on improving this paper.
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Average SINR Analysis of DS-BPSK UWB Systems With IPI, ICI, ISI, and MAI and Its Application

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Abstract—Analytical expressions for the average output signal-to-interference-plus-noise ratio (SINR) of direct-sequence (DS) binary-phase-shift-keying (BPSK) ultra-wideband (UWB) receivers are derived using the IEEE UWB channel models. The derivation takes into account the interpath interference, the interchip interference (IPI), the intersymbol interference (ISI), and the multiple-access interference (MAI). The effects of the spreading gain and the pulse duration on the average output SINR are examined. Using these results, the optimal integration interval for a transmitted reference UWB receiver is studied.

Index Terms—Average power, integration interval, signal-to-interference-plus-noise ratio (SINR), ultra-wide bandwidth.

I. INTRODUCTION

Among the properties of the received ultra-wideband (UWB) signals, the output signal-to-noise ratio (SNR) or the output signal-to-interference-plus-noise ratio (SINR) determines the performance of UWB systems [1]–[5]. In the future, the IEEE 802.15.3G3a study group released four UWB channel models in 2002 [6], many researchers studied the output SINR or the output SNR based on these channel models. In [7]–[9], the output SNR was evaluated by ignoring the interferences or by modeling the interferences as Gaussian. Closed-form expressions for the average output SNR were derived for Rake receivers in [10], and different SNR estimators were proposed in [11]. Furthermore, in [12], a closed-form expression for the channel-averaged output SINR of a Rake receiver was derived, based on a discrete-time channel model, where the multiple-access interference (MAI) was considered, but other interferences such as the interpath interference (IPI), the interchip interference (ICI), and the intersymbol interference (ISI) were ignored. The derivation is based on a constant delay spacing instead of the multipath delay. In [13], the SINR of a Rake receiver was analyzed by taking both ICI and MAI into account. All this work gives analytical expressions for estimating the SNR or the SINR for UWB systems. However, to the best of the authors’ knowledge, none of this work has taken all the necessary interferences in the channel into account. Due to the extremely large bandwidth of a UWB system, in addition to MAI and IPI, ICI and MAI may also occur in UWB systems. They also have significant effects on the SINR [14]. Considering this, it is of great interest to develop a framework that calculates the output SINR in a multiuser environment by taking IPI, ICI, MAI, and ISI into account.

In this paper, analytical expressions for the average output SINR of a direct-sequence (DS) BPSK UWB system are derived for the four IEEE UWB channel models by taking into account IPI, ICI, ISI, and MAI. Using these expressions, the effects of the length of the spreading code and the pulse duration on the average output SINR are examined. Numerical examples show that the sum of IPI, ICI, and ISI has a

Manuscript received July 16, 2008; revised December 4, 2008, February 10, 2009, and April 2, 2009. First published May 2, 2009; current version published October 2, 2009. This work was supported in part by the Engineering and Physical Sciences Research Council through the First Grant scheme under Grant EP/F030843/1. The review of this paper was coordinated by Prof. Y. Ma. The authors are with the School of Engineering, University of Warwick, CV4 7AL Coventry, U.K. (e-mail: Bo.Zhao@warwick.ac.uk; Yunfei.Chen@ warwick.ac.uk; Roger.Green@warwick.ac.uk).